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ASYMPTOTICS OF THREE-DIMENSIONAL MACROCRACK–MICROCRACK INTERACTION

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Abstract—The perturbation in the mode I stress intensity factor along the edge of a half-plane crack due to the opening of one or more co-planar penny-shaped cracks close to the edge calculated recently by the authors [*Int. J. Solids Structures* **30,** 2117–2139 (1993)] using the finite element method is shown to have the same logarithmic asymptotic behaviour as that of its plane strain two-dimensional counterpart. The approximate method of solution used in that paper, however, does not predict this asymptotic behaviour.

1. INTRODUCTION

In a recent paper (Huang and Karihaloo, 1993) the authors calculated the perturbation in the mode I stress intensity factor along the edge of a half-plane crack due to the opening of one or more co-planar penny-shaped cracks lying ahead of it. For this they used the three-dimensional weight functions (Rice, 1985; Bueckner, 1987; Karihaloo and Huang, 1989) and finite element method, after replacing the strong singularity in the weight functions by a weak one through the application of the Rayleigh–Ritz procedure. The finite element method provided a reliable solution even when the penny cracks were very close to the edge of the half-plane crack, but at the expense of significant computational time. To save on the latter, an approximate solution was also obtained on the assumption that the unknown opening displacement of each penny can be replaced by an unknown constant times a square-root function of its distance from the edge of the half-plane crack. This approximate solution was the better, the larger the separation between the penny and half-plane crack edge. It was also shown that the approximate solution coincided with that of Laures and Kachanov (1991), who regarded the average traction rather than the average opening displacement over the penny as the unknown.

In the present brief paper we re-examine the finite element solution obtained in that earlier paper (Huang and Karihaloo, 1993) with a view to gaining a deeper understanding of the asymptotic behaviour of the mode I stress intensity factor along the edge of a half-plane crack due to the opening of one or more co-planar penny cracks lying close to it. It is shown that this behaviour is identical to the logarithmic dependence at the tip of a semi-infinite crack with a finite co-planar crack lying close to it. This behaviour is not, however, predicted by the approximate solution given by Huang and Karihaloo (1993). Consequently, by inference, it cannot also be predicted by the solution given by Laures and Kachanov (1991).

2. FINITE ELEMENT RESULTS

For the purposes of this paper, we have re-calculated using the finite element method the stress intensity factor K_I along the edge of a half-plane crack due to its interaction with a single co-planar penny crack, shown in the inset of Fig. 1. The stress intensity factor K_I is normalized with respect to the stress intensity factor K_I^∞ due to remotely applied mode I loading in the absence of a penny crack, so that $(K_I/K_I^\infty)-1$ represents the perturbation due to the opening of the penny crack. In the earlier paper (Huang and Karihaloo, 1993) the authors had calculated only the maximum values of K_I/K_I^∞ using the finite element method, whereas the variation of this ratio with $|z|/b$ (Fig. 8 of that paper) was calculated

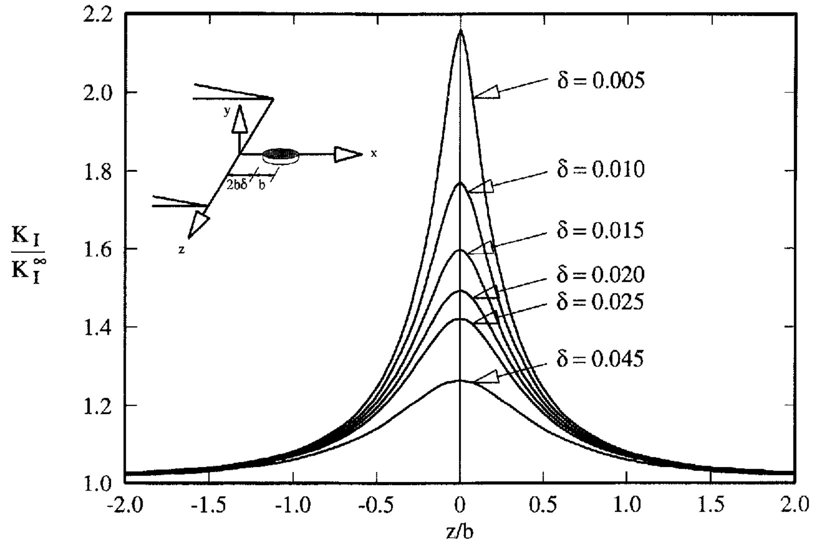

Fig. 1. Variation in K_I/K_I^∞ along the edge of a half-plane crack due to a co-planar penny crack.

using the approximate method. Figure 1 therefore complements Fig. 8 from that paper. Table 1 gives the maximum of K_I/K_I^∞ for several small values of the distance between the edge of the half-plane crack and the pole of the penny closest to the edge (Fig. 1). These values differ slightly from the corresponding entries in the last column of Table 1 from our earlier paper because of the use of a finer finite element mesh.

3. ASYMPTOTIC BEHAVIOUR AS $\delta \to 0$

Before studying the asymptotic behaviour of K_I/K_I^∞ as $\delta \to 0$, it is interesting to observe the bell-shaped variation of K_I/K_I^∞ and to quantify the rate of its decay with increasing $|z|/b$. The deviation from unity is already less than 10% (Fig. 1) at a distance along the edge of the half-plane crack equal to the radius of the penny ($|z|/b = 1$). Numerical results show that the decay in the deviation from unity can be faithfully represented by

$$\frac{K_I}{K_I^\infty} - 1 = \frac{A(\delta)}{(z^*)^2 + B(\delta)}, \tag{1}$$

where $z^* = z/b$, and for small δ in the range $0.005 \leqslant \delta \leqslant 0.045$

$$A(\delta) = 0.0975 - 0.2618\delta, \quad B(\delta) = 0.0763 + 5.9714\delta. \tag{2}$$

Table 1. Maximum values of K_I/K_I^∞ (at $z = 0$) along the edge of a half-plane crack due to its interaction with a co-planar penny crack of radius b centred at $(x_0, 0, 0)$, with $x_0 = b(2\delta + 1)$

δ	$(\text{FEM})_{\text{single}}$
0.005	2.1576
0.010	1.7696
0.015	1.5983
0.020	1.4938
0.025	1.4213
0.045	1.2643

The maximum deviation at $z = 0$ is given by

$$\max\left(\frac{K_\mathrm{I}}{K_\mathrm{I}^\infty} - 1\right) \equiv M(\delta) = \frac{A(\delta)}{B(\delta)}. \tag{3}$$

In view of the rapid decay in the deviation of $K_\mathrm{I}/K_\mathrm{I}^\infty$ from unity with increasing $(|z|/b)$, one may ignore the mutual interaction of non-overlapping penny cracks in an infinite array (Fig. 2) and write by inspection

$$\frac{K_\mathrm{I}}{K_\mathrm{I}^\infty} - 1 = \sum_{\lambda=-\infty}^{\infty} \frac{A(\delta)}{B(\delta) + (z^* + \lambda z_0^*)^2}, \tag{4}$$

where $z_0^* = z_0/b$. The standard sum in eqn (4) can be evaluated to give

$$\frac{K_\mathrm{I}}{K_\mathrm{I}^\infty} - 1 = \frac{A(\delta)}{\sqrt{B(\delta)}} \frac{\pi}{z_0^*} \frac{\sinh(2\pi\sqrt{B(\delta)}/z_0^*)}{\cosh(2\pi\sqrt{B(\delta)}/z_0^*) - \cos(2\pi z^*/z_0^*)}. \tag{5}$$

For a given spacing between the penny cracks z_0^*, the maxima of the deviation from unity occur at $z^* = \pm n(n = 0, 1, 2, \ldots)$, i.e. at the locations along the edge of half-plane crack that correspond to the centres of penny cracks. In particular, at the location $z^* = 0$,

$$\max\left(\frac{K_\mathrm{I}}{K_\mathrm{I}^\infty} - 1\right) \equiv M_{\mathrm{row}}(\delta) = \frac{A(\delta)}{\sqrt{B(\delta)}} \frac{\pi}{z_0^*} \frac{\sinh(2\pi\sqrt{B(\delta)}/z_0^*)}{\cosh(2\pi\sqrt{B(\delta)}/z_0^*) - 1}. \tag{6}$$

A comparison of $M_{\mathrm{row}}(\delta)$ from eqn (6) with $M(\delta)$ for a single penny from eqn (3) shows that the maximum deviation of $K_\mathrm{I}/K_\mathrm{I}^\infty$ from unity in the two configurations differs by the factor

$$\frac{M_{\mathrm{row}}(\delta)}{M(\delta)} \equiv C(\delta) = \frac{\pi}{z_0^*}\sqrt{B(\delta)} \frac{\sinh(2\pi\sqrt{B(\delta)}/z_0^*)}{\cosh(2\pi\sqrt{B(\delta)}/z_0^*) - 1}. \tag{7}$$

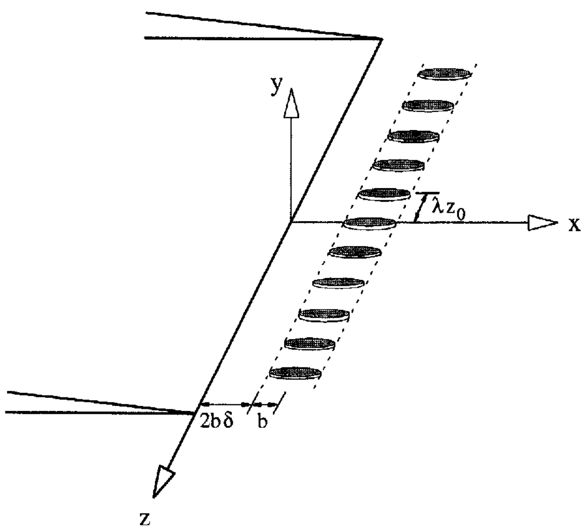

Fig. 2. A half-plane crack interacting with a periodic array of non-overlapping co-planar penny cracks.

Table 2. Maximum values of K_I/K_I^∞ (at $z = 0$) along the edge of a half-plane crack due to its interaction with an infinite array of co-planar penny cracks each of radius b centred at $(x_0,\ 0,\ |z_0|)$, with $x_0 = b(2\delta+1)$, $|z_0| = 2nb(n = 0,1,2,\ldots)$

δ	$B(\delta)$	$C(\delta)$	$(\mathrm{FEM})_{\mathrm{row}}$
0.005	0.1062	1.0859	2.2570
0.010	0.1360	1.0912	1.8398
0.015	0.1659	1.1329	1.6778
0.020	0.1957	1.1560	1.5708
0.025	0.2256	1.1790	1.4967
0.045	0.3450	1.2688	1.3353

Formula (7) can be rewritten as

$$C(\delta) = u(\delta)\coth u(\delta), \tag{8}$$

where $u(\delta) = \pi\sqrt{B(\delta)}/z_0^*$. For small $u(\delta)$

$$C(\delta) \sim 1 + u^2(\delta)/3. \tag{9}$$

Using eqn (7) or (8), we can immediately construct from Table 1 the ratio of maximum K_I/K_I^∞ for an infinite row of non-overlapping penny cracks, i.e.

$$(\mathrm{FEM})_{\mathrm{row}} = [(\mathrm{FEM})_{\mathrm{single}} - 1]C(\delta) + 1.$$

The results are given in Table 2 for the limiting case of touching pennies, i.e. $z_0^* = 2$.

Next, we replace the infinite row of touching penny cracks by a strip $2b\delta < x < 2b(1+\delta)$ (Fig. 3a), so that the configuration of half-plane crack and co-planar

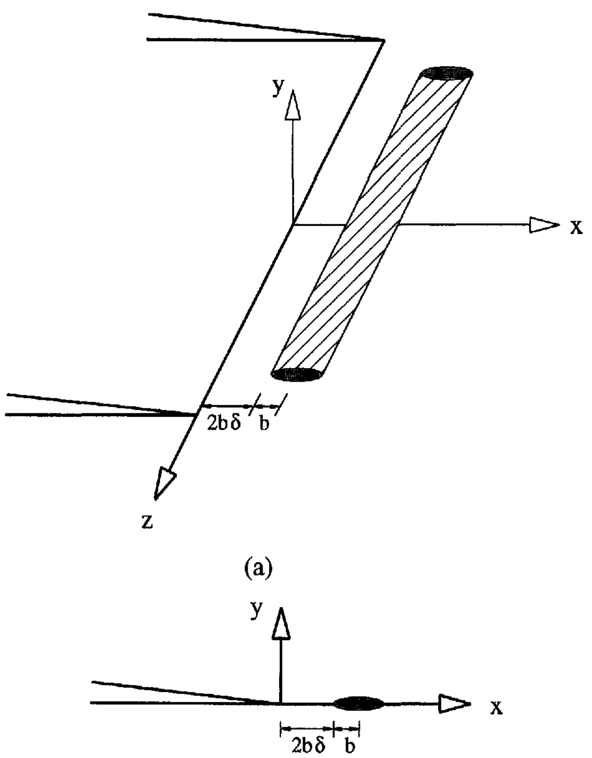

Fig. 3. The configuration of a half-plane crack interacting with a co-planar cracked strip (a) analysed as one of plane strain for a semi-infinite crack interacting with a co-planar finite crack (b).

Table 3. Product of $f(\delta) = \sqrt{\delta}\log(1/\delta)$ and finite element results for the maximum K_I/K_I^∞ (at $z = 0$) along the edge of a half-plane crack interacting with a single co-planar penny crack or an infinite row of co-planar touching pennies

δ	$f(\delta)$	$f(\delta)(\text{FEM})_{\text{single}}$	$f(\delta)(\text{FEM})_{\text{row}}$
0.005	0.3746	0.8082	0.8455
0.010	0.4605	0.8149	0.8472
0.015	0.5144	0.8222	0.8631
0.020	0.5532	0.8263	0.8690
0.025	0.5833	0.8290	0.8730
0.045	0.6578	0.8317	0.8784

strip can be analysed as one of plane strain (Fig. 3b). For this configuration, the stress intensity factor K_I at the tip of the semi-infinite crack due to the opening of the co-planar finite crack is known in a closed form (Rubinstein, 1985, 1994; Rose, 1986)

$$\frac{K_I}{K_I^\infty} = \sqrt{\frac{1}{\delta}}\frac{E(1-\delta)}{K(1-\delta)}, \tag{10}$$

where $E()$ and $K()$ are elliptic functions. Using their asymptotic properties as $\delta \to 0$, one finds that

$$\lim_{\delta \to 0}\frac{K_I}{K_I^\infty} \sim \frac{2}{\sqrt{\delta}\log(1/\delta)}. \tag{11}$$

The solution (11) can also be obtained in terms of hypergeometric functions using the weight functions for the configuration of collinear cracks given by Bueckner (1975).

Let us now construct the product of $f(\delta) = \sqrt{\delta}\log(1/\delta)$ and finite element results for K_I/K_I^∞ for a single penny and an infinite row of touching pennies using the entries in the last column of Tables 1 and 2. The results are given in Table 3.

The mean value of the product for a single penny is 0.8221 (standard deviation 0.9%) and for an array of touching pennies is 0.8627 (standard deviation 1.36%). These values demonstrate that with an error of less than 1.5% the maximum values of K_I/K_I^∞ (at $z = 0$) along the edge of a half-plane crack interacting with a single co-planar penny crack or an infinite row of co-planar touching pennies exhibit the same asymptotic behaviour (as $\delta \to 0$) as K_I/K_I^∞ at the tip of a semi-infinite crack interacting with a finite co-planar crack. Moreover, the maximum values for the two three-dimensional configurations differ asymptotically by a constant factor equal to 0.8627/0.8221 = 1.049. The knowledge of this asymptotic behaviour can be highly useful in studying other three-dimensional macro–microcrack interactions.

However, the maximum values of K_I/K_I^∞ obtained using the approximate methods of Huang and Karihaloo (1993) and of Laures and Kachanov (1991) do not predict this asymptotic behaviour as $\delta \to 0$. This may be readily verified by forming the product of $f(\delta)$ with entries from columns two and three of Table 1 in the work of Huang and Karihaloo (1993) and observing that this product does not tend to a constant as $\delta \to 0$. It would therefore seem that the approximate methods of three-dimensional macrocrack–microcrack interactions proposed by Laures and Kachanov (1991) and by Huang and Karihaloo (1993) are accurate only when the microcracks are well removed from the macrocrack edge. When the microcracks are close to the edge of the macrocrack, eqns (1) and (5) above would give an accurate estimate of the perturbation in the stress intensity factor K_I.

Acknowledgements—The motivation for this brief asymptotic exploration was provided by the incisive comments of the reviewer on our earlier paper (Huang and Karihaloo, 1993), who had correctly anticipated some of the asymptotic results reported above. It would have been most appropriate for that reviewer to appear as the first author of this paper, but as we are not privy to his/her identity, we can only offer our sincere thanks. We are also grateful to Professor A. A. Rubinstein and Dr L. R. F. Rose for many useful discussions.

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